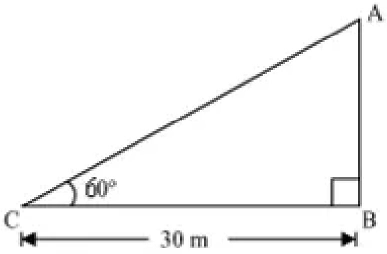
Chapter 22. Heights and Distances

Ex 22.1

Answer 2.



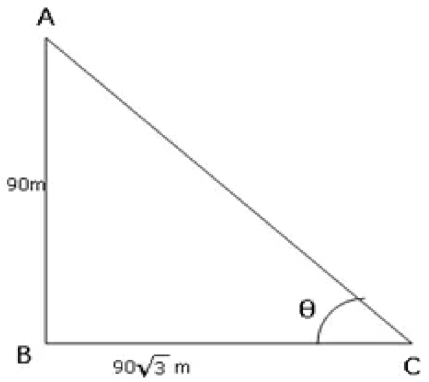
Let AB be the cliff and angle of elevation from point C (on ground) is 30 °.

In ΔABC

$$\frac{AB}{BC} = \tan 60^\circ$$
$$\frac{AB}{30} = \sqrt{3}$$
$$AB = 30\sqrt{3} \text{ m}$$

So, height of tower is $30\sqrt{3}$ m.

Answer 3.



Let AB be the pole and BC be its shadow.

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

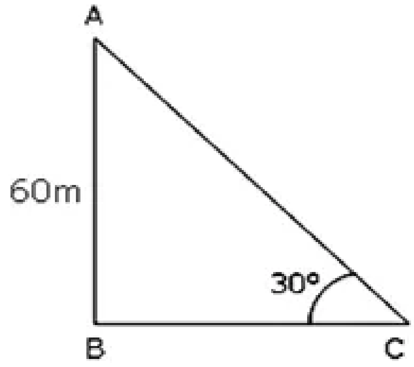
$$\Rightarrow \tan\theta = \frac{90}{90\sqrt{3}} = \frac{1}{\sqrt{3}}$$

But, $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$

Thus, the angle of elevation is 30°.

Answer 5.



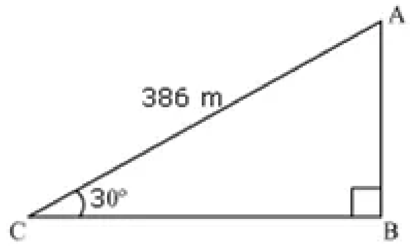
Let AB be the tree of height 60 m and BC be its shadow.

In ΔABC

$$\frac{AB}{BC} = \tan 30^\circ$$
$$\frac{60}{BC} = \frac{1}{\sqrt{3}}$$
$$BC = 60\sqrt{3} \text{ m}$$

So, height of tower is $60\sqrt{3}$ m.

Answer 6.



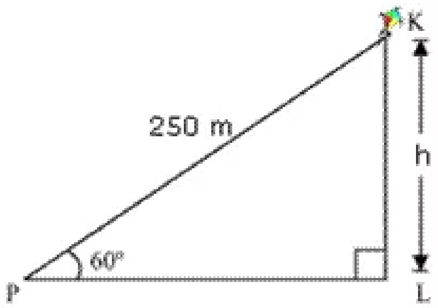
The plane takes off from point C on the ground. Let A be the final position of the plane.

In ΔABC

$$\frac{AB}{AC} = \sin 30^\circ$$
$$\frac{AB}{386} = \frac{1}{2}$$
$$h = \frac{386}{2} = 193$$

Thus, the required height of the aeroplane above the ground is 193 m.

Answer 7.



Let K be the kite and the string is tied to point P on ground.

In ΔKLP

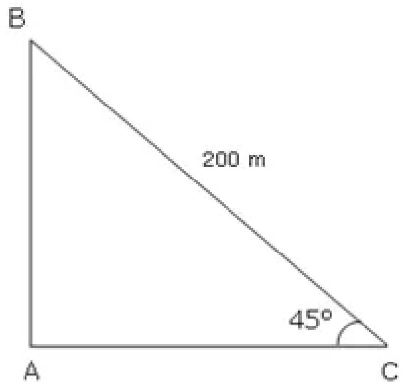
$$\frac{KL}{KP} = \sin 60°$$

$$\frac{h}{250} = \frac{\sqrt{3}}{2}$$

$$h = \frac{250\sqrt{3}}{2} = 125\sqrt{3}$$

Thus, the perpendicular height of the kite is $125\sqrt{3}$ m.

Answer 8.



The topmost branch of the tree is at point B and C is the point on the ground to which the topmost branch is tied.

In ΔABC,

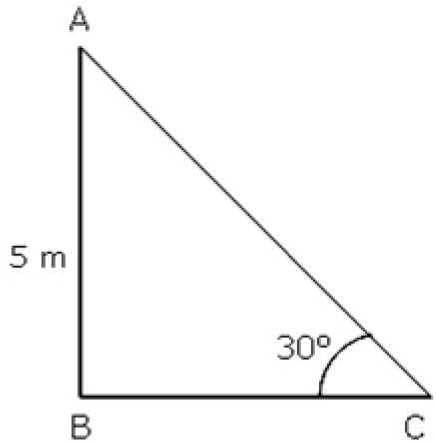
$$\frac{AC}{BC} = \cos 45°$$

$$\frac{h}{200} = \frac{1}{\sqrt{2}}$$

$$h = \frac{200}{\sqrt{2}} = \frac{200\sqrt{2}}{2} = 100\sqrt{2} = 100 \times 1.414 = 141.4$$

Thus, the required distance is 141.4 m.

Answer 9.



Here, AC is the ladder and AB is the point which is 5 m above the ground.

In ΔABC,

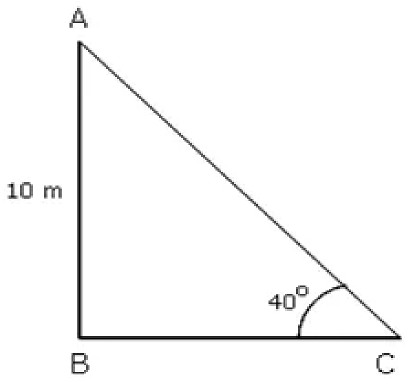
$$\sin 30^\circ = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{5}{AC}$$

$$\Rightarrow AC = 10$$

Thus, the length of the ladder is 10 m.

Answer 10.



Let AB be the pole and AC be the wire which runs from the top of the pole to the point on the ground where its other end is fixed.

In ΔABC,

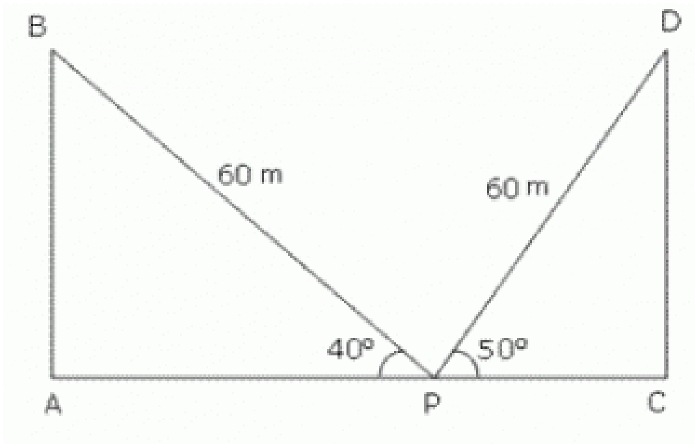
$$\sin 40^\circ = \frac{AB}{AC}$$

$$\Rightarrow 0.6428 = \frac{10}{AC}$$

$$\Rightarrow AC = \frac{10}{0.6428} = 15.6$$

Thus, the length of the wire is 15.6 m.

Answer 11.



Let AB and CD be two trees and P be a point on the street AC between th two trees.

PD and PB denotes the ladder at the two instants.

In ΔPCD,

$$\cos 50^\circ = \frac{PC}{PD}$$
$$0.6428 = \frac{PC}{60}$$
$$\Rightarrow PC = 0.6428 \times 60 = 38.568$$

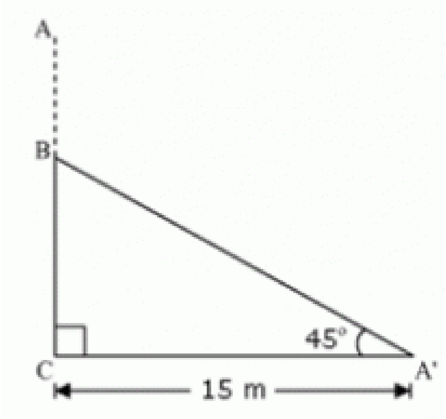
In ΔABP,

$$\cos 40^\circ = \frac{AP}{BP}$$
$$\Rightarrow 0.7660 = \frac{AP}{60}$$
$$\Rightarrow AP = 0.7660 \times 60 = 45.96$$

$\therefore AC = AP + PC = 38.568\text{ m} + 45.96\text{m} = 84.528\text{ m} \approx 84.53\text{ m}.$

Thus, the width of the street is 84.53 m.

Answer 12



Let AC was original tree. Due to storm it was broken into two parts. Th
broken part A' B is making 45° with ground.

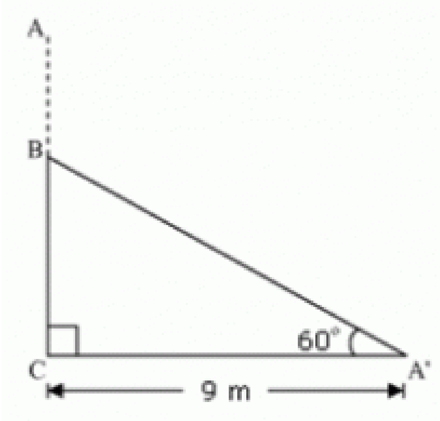
In $\Delta A'BC$

$$\frac{BC}{A'C} = \tan 45^\circ$$
$$\frac{BC}{15} = 1$$
$$BC = 15$$
$$\frac{A'C}{A'B} = \cos 45^\circ$$
$$\frac{15}{A'B} = \frac{1}{\sqrt{2}}$$
$$A'B = 15\sqrt{2}$$

Height of tree = $A'B + BC = 15 + 15\sqrt{2} = 15(1+\sqrt{2}) = 15 \times 2.414 = 36.21$

Hence, the height of tree was 36.21 m.

Answer 13.



Let AC was original tree. It was broken into two parts. The broken pa
A' B is making 60° with ground.

In $\Delta A'BC$

$$\frac{BC}{A'C} = \tan 60^\circ$$

$$\frac{BC}{9} = \sqrt{3}$$

$$BC = 9\sqrt{3}$$

$$\frac{A'C}{A'B} = \cos 60^\circ$$

$$\frac{9}{A'B} = \frac{1}{2}$$

$$A'B = 18$$

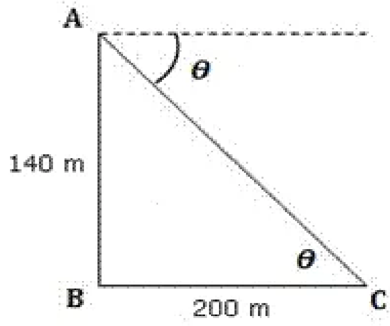
Height of tree = $A'B + BC$

$= 9\sqrt{3} + 18 = 9 \times 1.732 + 18 = 15.588 + 18 = 33.588$

Hence, the height of tree was 33.588 m = 33.6 m (approximately).

Answer 14.

Let AB be the pillar. Let the angle of depression be θ.



In triangle ABC,

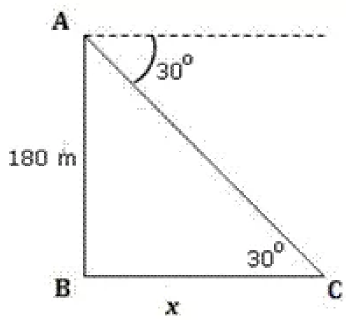
$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \tan\theta = \frac{140}{200} = \frac{7}{10} = 0.7$$

We have: tan 35° = 0.7

Thus, the angle of depression is θ = 35°.

Answer 15.



Let AB be the tower and C be the position of the ship.

Let the distance of the boat from the foot of the observation tower be x.

In triangle ABC,

$$\tan\theta = \frac{AB}{BC}$$

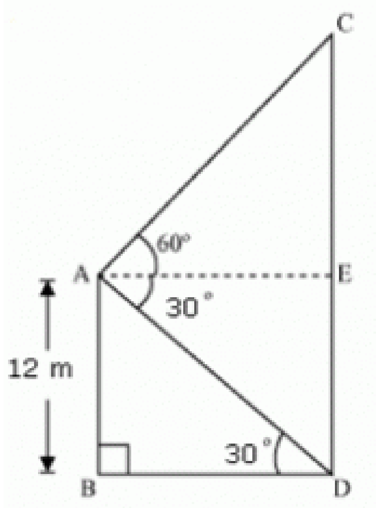
$$\Rightarrow \tan 30^\circ = \frac{180}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{180}{x}$$

$$\Rightarrow x = 180\sqrt{3} = 180 \times 1.732 = 311.76$$

Thus, the distance of the boat from the foot of the observation tower is 311.76 or 311.8 m.

Answer 16.



Let AB be the building and CD be the tower.

In ΔABD

$$\frac{AB}{BD} = \tan 30^\circ$$
$$\frac{12}{BD} = \frac{1}{\sqrt{3}}$$
$$BD = 12\sqrt{3}$$

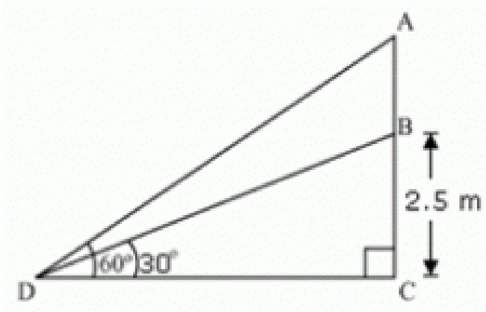
In ΔACE

$AE = BD = 12\sqrt{3}$

$$\frac{CE}{AE} = \tan 60^\circ$$
$$\frac{CE}{12\sqrt{3}} = \sqrt{3}$$
$$CE = 12\sqrt{3} \times \sqrt{3} = 12 \times 3 = 36$$
$$CD = CE + ED = 36 + 12 = 48$$

So, height of the tower is 48 m and its distance from the building $12\sqrt{3}$ m = 12 x 1.732 m= 20.78 m(approximately).

Answer 17.



Let AB be the flagstaff, BC be the pole and D be the point on ground frc where elevation angles are measured.

In ΔBCD

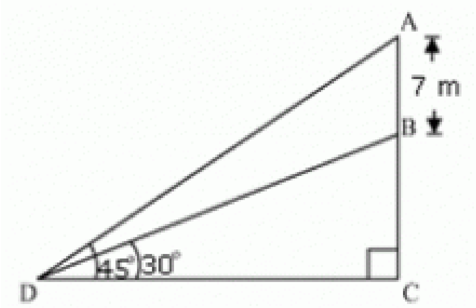
$$\frac{BC}{CD} = \tan 30^\circ$$
$$\frac{BC}{CD} = \frac{1}{\sqrt{3}}$$
$$\sqrt{3}BC = CD \qquad \ldots (1)$$

In ΔACD

$$\frac{AB + BC}{CD} = \tan 60^\circ$$
$$\frac{AB + BC}{CD} = \sqrt{3}$$
$$AB + 2.5 = CD\sqrt{3} = 3BC \qquad [\text{Using } (1)]$$
$$AB + 2.5 = 3 \times 2.5$$
$$AB + 2.5 = 7.5$$
$$AB = 5$$

Thus, the height of the flagstaff is 5 m.

Answer 18.



Let AB be the flagstaff, BC be the tower and D be the point on grou from where elevation angles are measured.

In ΔBCD

$$\frac{BC}{CD} = \tan 30^\circ$$

$$\frac{BC}{CD} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}BC = CD$$

In ΔACD

$$\frac{AB + BC}{CD} = \tan 45^\circ$$

$$\frac{AB + BC}{\sqrt{3}BC} = 1$$

$$7 + BC = \sqrt{3}BC$$

$$BC\left(\sqrt{3} - 1\right) = 7$$

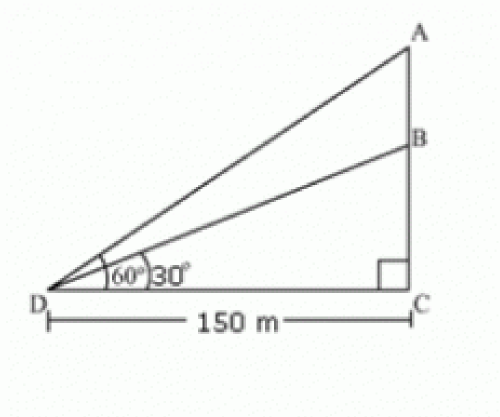
$$BC = \frac{(7)\left(\sqrt{3} + 1\right)}{\left(\sqrt{3} - 1\right)\left(\sqrt{3} + 1\right)}$$

$$= \frac{7\left(\sqrt{3} + 1\right)}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{7\left(\sqrt{3} + 1\right)}{2} = 3.5\left(\sqrt{3} + 1\right) = 3.5 \times 2.732 = 9.562$$

Thus, the height of the tower is 9.562 m = 9.56 m.

Answer 19.



Let BC be the length of unfinished tower. Let the tower be raised up point A so that the angle of elevation at point A is 60°. D is the point ground from where elevation angles are measured.

In ΔBCD

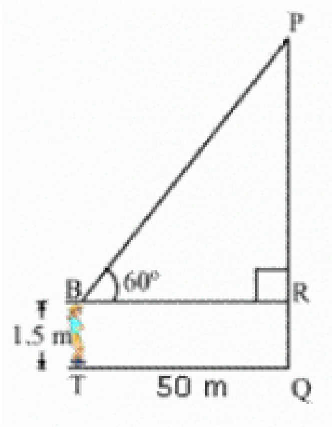
$$\frac{BC}{CD} = \tan 30°$$
$$\frac{BC}{CD} = \frac{1}{\sqrt{3}}$$
$$BC = \frac{CD}{\sqrt{3}}$$
$$BC = \frac{150}{\sqrt{3}} \qquad \ldots(1)$$

In ΔACD

$$\frac{AB + BC}{CD} = \tan 60°$$
$$\Rightarrow \frac{AB + BC}{CD} = \sqrt{3}$$
$$\Rightarrow \frac{AB + \frac{150}{\sqrt{3}}}{150} = \sqrt{3} \quad \ldots.(\text{using}(i))$$
$$\Rightarrow AB = 150\sqrt{3} - \frac{150}{\sqrt{3}} = \frac{150 \times 3 - 150}{\sqrt{3}}$$
$$\Rightarrow AB = \frac{300}{\sqrt{3}} = \frac{300}{1.732} = 173.2m$$

Thus, the required height is 300 m.

Answer 20.



Let the position of the boy be at point T.

BR = TQ = 50 m

RQ = BT = 1.5 m

In ΔPRB

$$\frac{PR}{BR} = \tan 60^\circ$$

$$\frac{PR}{50} = \sqrt{3}$$

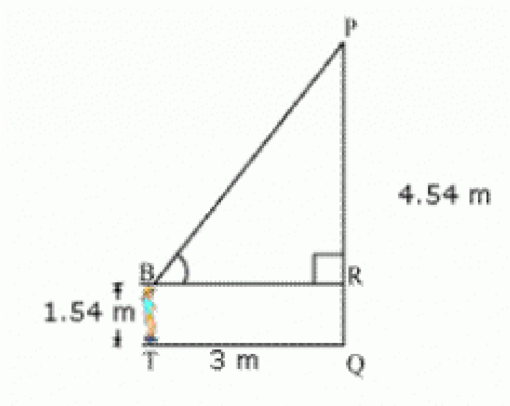
$$PR = 50\sqrt{3}$$

Height of the tower

$= PQ = PR + RQ = 50\sqrt{3} + 1.5 = 50 \times 1.732 + 1.5 = 86.6 + 1.5 = 88.1$ m

Thus, the height of the tower is approximately 88 m.

Answer 21.



Let the position of the boy be at point T and P be the position of the sur

BR = TQ = 3 m

PQ = 4.54 m

BT = 1.54 m

$\therefore$ PR = 4.54 m – 1.54 m = 3 m

In ΔPRB

$$\frac{PR}{BR} = \tan\theta$$

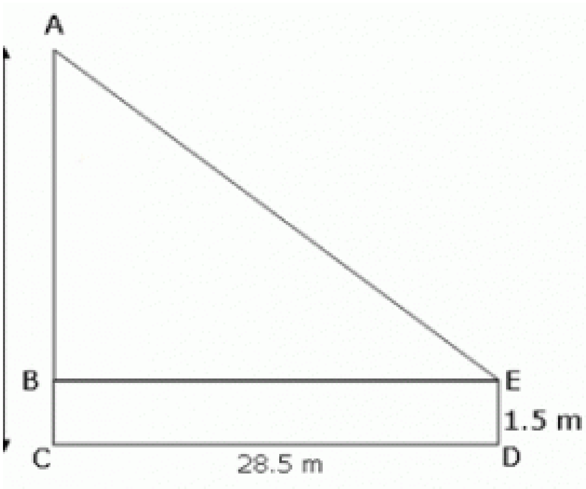
$$\frac{3}{3} = \tan\theta$$

$$\tan\theta = 1$$

We know that tan 45° = 1.

Thus, the angle of elevation is $\theta = 45°$.

Answer 22.



Here, ED is the height of the observer and AC is the tower.

BE = CD = 28.5 m

AB = AC – BC = 30 m – 1.5 m = 28.5 m

In ΔABE,

$$\tan \angle ABE = \frac{AB}{BE}$$

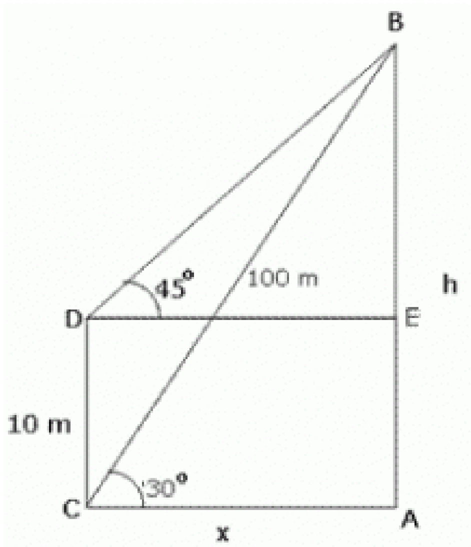
$$\Rightarrow \tan \angle ABE = \frac{28.5m}{28.5m} = 1$$

But, $\tan 45^\circ = 1$

$\therefore \angle ABE = 45^\circ$

Thus, the required angle of elevation is 45°.

Answer 23.



Let C be the position of the first boy and D be the position of the secor boy who is standing on the roof of a 10 m high building.

Let B be the position of the kites of both the boys.

Let AB = h and CA = x.

In ΔABC,

$$\sin 30^\circ = \frac{h}{100}$$
$$\Rightarrow \frac{1}{2} = \frac{h}{100}$$
$$\Rightarrow h = 50 \qquad ...(1)$$

In ΔBDE,

$$\tan 45^\circ = \frac{BE}{BD}$$
$$\Rightarrow 1 = \frac{h-10}{x}$$
$$\Rightarrow x = (h-10) \qquad ...(2)$$

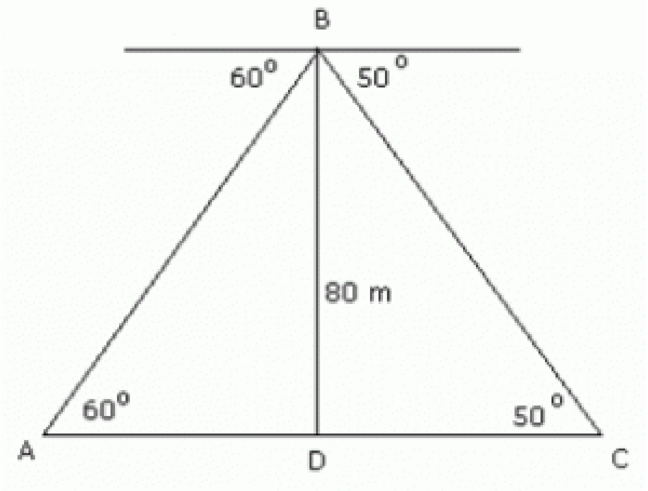
From (1) and (2),

x = 50 − 10 = 40

$$\sin 45^\circ = \frac{BE}{BD}$$
$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{h-10}{BC}$$
$$\Rightarrow BC = \sqrt{2}(50-10) = 40\sqrt{2}$$

Thus, the required length of the string that the second boy must have $40\sqrt{2}$ m

Answer 25.



Let the position of the two persons be A and C. Let BD be the tower height 80 m.

In ΔBAD,

$$\tan 60^\circ = \frac{BD}{AD}$$
$$\Rightarrow \sqrt{3} = \frac{80}{AD}$$
$$\Rightarrow AD = \frac{80}{\sqrt{3}}$$
$$\Rightarrow AD = \frac{80\sqrt{3}}{3} = \frac{80 \times 1.732}{3} = 46.19 \qquad \text{...(1)}$$

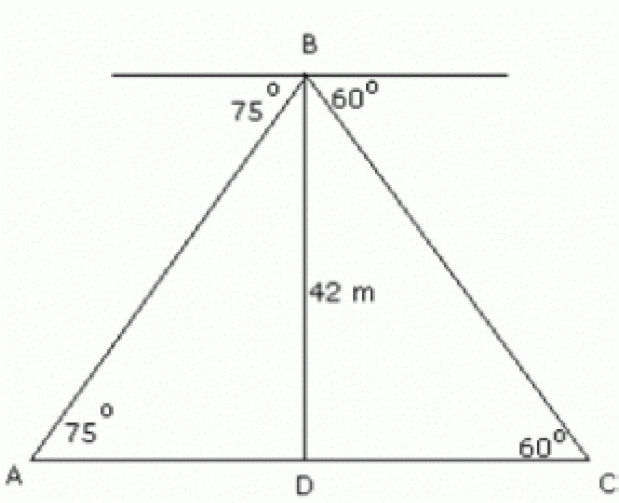
In ΔBDC,

$$\tan 50^\circ = \frac{BD}{DC}$$
$$\Rightarrow 1.1918 = \frac{80}{DC}$$
$$\Rightarrow DC = \frac{80}{1.1918} = 67.13 \qquad \text{...(2)}$$

$\therefore AC = AD + DC = 46.19 \text{ m} + 67.13 \text{ m} = 113.32 \text{ m}$

Thus, the horizontal distance between the two persons is 113.32 m.

Answer 26.



Let the position of the two cars be A and C. Let BD be the building height 42 m.

In ΔBAD,

$$\tan 75^\circ = \frac{BD}{AD}$$
$$\Rightarrow 3.7321 = \frac{42}{AD}$$
$$\Rightarrow AD = \frac{42}{3.7321}$$
$$\Rightarrow AD = 11.25 \qquad ...(1)$$

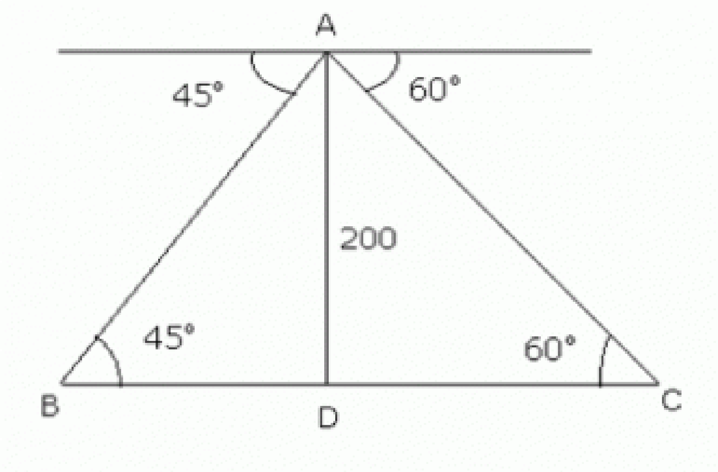
In ΔBDC,

$$\tan 60^\circ = \frac{BD}{DC}$$
$$\Rightarrow \sqrt{3} = \frac{42}{DC}$$
$$\Rightarrow DC = \frac{42}{1.732} = 24.25 \qquad ...(2)$$

$\therefore AC = AD + DC = 11.25 \text{ m} + 24.25 \text{ m} = 35.5 \text{ m}$

Thus, the distance between the cars is 67.63 m.

Answer 27.



Let AD be the height of the aeroplane and BC = x m be the width of the river.

Given: AD=200m

In ΔABD

$$\frac{AD}{BD} = \tan 45^\circ$$

$$\Rightarrow \frac{AD}{BD} = 1$$

$$\Rightarrow AD = BD$$

$$\Rightarrow BD = 200m \ (\because AD = 200m)$$

Now,

In ΔACD

$$\frac{AC}{CD} = \tan 60^\circ$$

$$\Rightarrow \frac{AC}{CD} = \sqrt{3}$$

$$\Rightarrow CD = \frac{AC}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

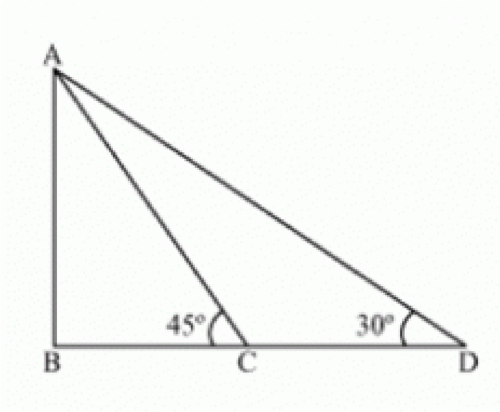
$$\Rightarrow BC = BD + CD = 200 + \frac{200}{\sqrt{3}} = 200 + 115.47$$

$$\Rightarrow BC = 315.4m$$

Thus, the width of the river is 315.4 m.

Answer 28.

Case 1: When the boats are on same side of the observation point.



Let the position of the two ships be C and D. Let A be the point of observation.

AB = 500 m

In ΔBAC,

$$\tan 45^\circ = \frac{AB}{BC}$$
$$\Rightarrow 1 = \frac{500}{BC}$$
$$\Rightarrow BC = 500 \qquad ...(1)$$

In ΔABD,

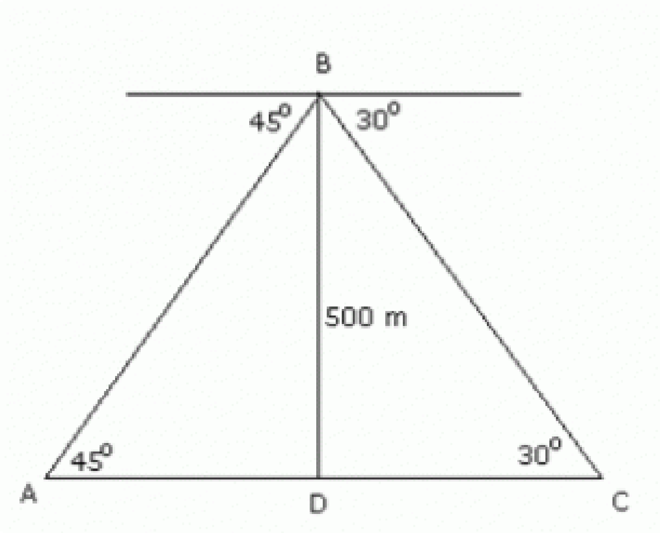
$$\tan 30^\circ = \frac{AB}{BD}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{500}{BD}$$
$$\Rightarrow BD = 500\sqrt{3} \qquad ...(2)$$

From (1) and (2),

$CD = BD - BC = 500(\sqrt{3} - 1) = 500 \times 0.732 = 366$

Thus, in this case, the distance between the boats is 366 m.

Case 2: When the boats are on different side of the observation point.



Let the position of the two ships be A and C. Let B be the point of observation.

In ΔBAD,

$$\tan 45° = \frac{BD}{AD}$$
$$\Rightarrow 1 = \frac{500}{AD}$$
$$\Rightarrow AD = 500 \qquad \ldots(1)$$

In ΔBDC,

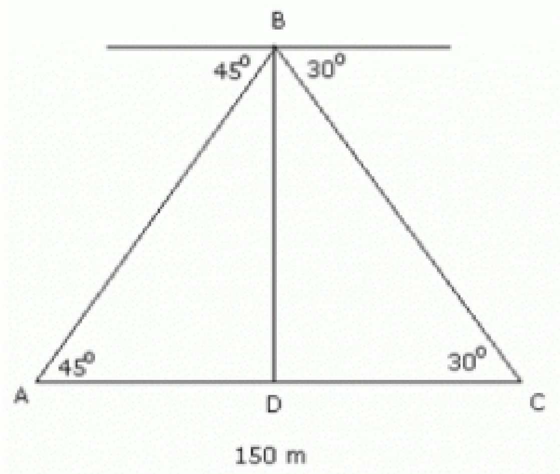
$$\tan 30° = \frac{BD}{DC}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{500}{DC}$$
$$\Rightarrow DC = 500\sqrt{3} \qquad \ldots(2)$$

From (1) and (2),

$AC = AD + DC = 500\,(1 + \sqrt{3}) = 500 \times 2.732 = 1366$

Thus, in this case, the distance between the boats is 1366 m.

Answer 29.



Let the position of the two boats be at points A and C. Let BD be t lighthouse of height h.

Let AD = x. Then, CD = 150 − x

In ΔBAD,

$$\tan 45^\circ = \frac{BD}{AD}$$
$$\Rightarrow 1 = \frac{h}{x}$$
$$\Rightarrow h = x \qquad ...(1)$$

In ΔBDC,

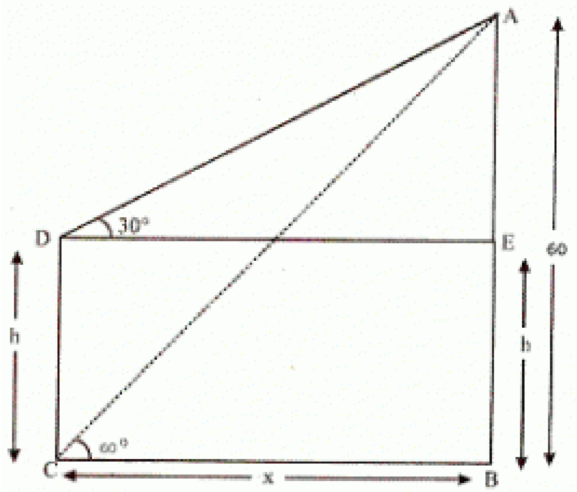
$$\tan 30^\circ = \frac{BD}{DC}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{150 - x}$$
$$\Rightarrow 150 - x = \sqrt{3}h \qquad ...(2)$$

From (1) and (2),

$$150 - h = \sqrt{3}h$$
$$150 = (\sqrt{3} + 1)h$$
$$h = \frac{150}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= \frac{150(\sqrt{3}-1)}{3-1}$$
$$= 75(\sqrt{3}-1)$$
$$= 75 \times 0.732 = 54.9$$

Thus, the height of the light house is 54.9 m.

Answer 30.



Let AB be the building. Then, AB = 60 m.

Let the height of the lamp post (CD) be h.

Let the distance between the building and the lamp post be x.

In ΔACB,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 20 \times 1.732 = 34.64 \ldots (1)$$

Thus, the distance between the building and the lamp post is 34.64 m.

In ΔADE,

$$\tan 30° = \frac{AE}{DE}$$

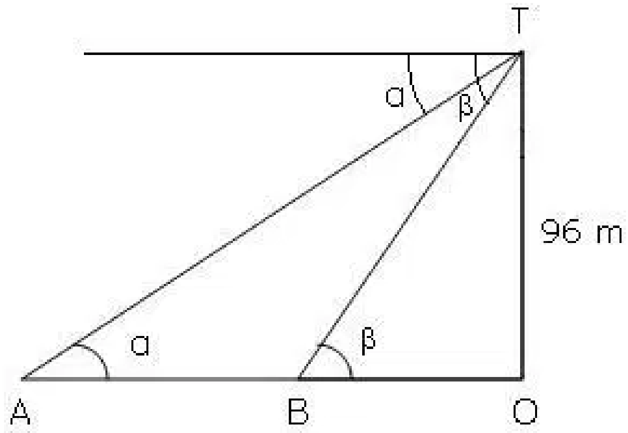
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60 - h}{x}$$

$$\Rightarrow x = \sqrt{3}(60 - h) \quad \ldots (2)$$

From (1) and (2):

$$\sqrt{3}(60 - h) = 20\sqrt{3}$$
$$60 - h = 20$$
$$h = 40$$

Answer 31.



In the figure, TO is the light house and A and B are the position of the two ships.

In ΔAOT,

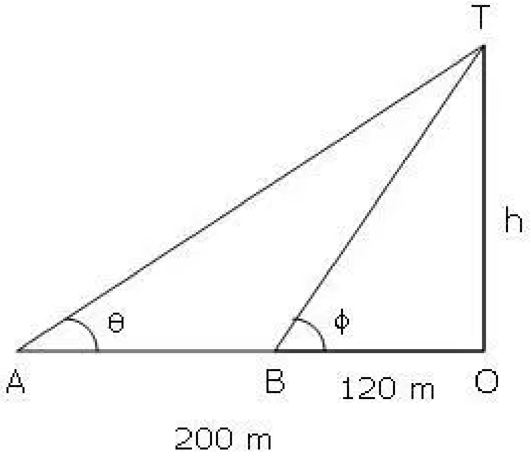
$$\frac{OT}{OA} = \tan\alpha$$
$$\Rightarrow \frac{96}{OA} = \frac{1}{4}$$
$$\Rightarrow OA = 384$$

In ΔBOT,

$$\frac{OT}{OB} = \tan\beta$$
$$\Rightarrow \frac{96}{OB} = \frac{1}{7}$$
$$\Rightarrow OB = 672$$

$\therefore$ Distance between the two ships = AB = OA – OB = 384 – 672 = 288m

Answer 32.



Let OT be the tower.

A and B be the two points from where the angle of elevation to the top of the tower is measured.

In ΔAOT,

$$\frac{OT}{OA} = \tan\theta$$

$$\Rightarrow \frac{h}{200} = \frac{2}{5}$$

$$\Rightarrow h = 80 \qquad \ldots(1)$$

Thus, the height of the tower is 80 m.

In ΔBOT,

$$\frac{OT}{OB} = \tan\phi$$

$$\Rightarrow \frac{h}{120} = \tan\phi$$

$$\Rightarrow \frac{80}{120} = \tan\phi \qquad \text{[Using (1)]}$$

$$\Rightarrow \frac{2}{3} = \tan\phi$$

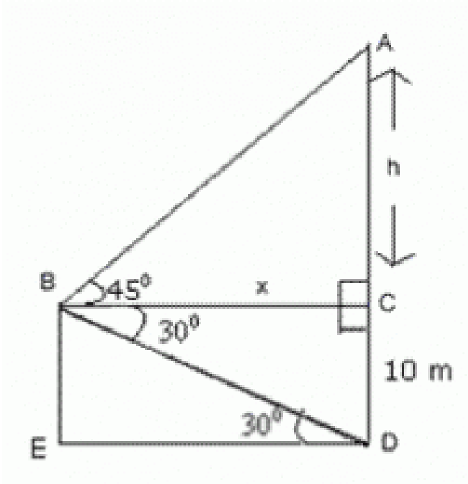
From the table, we get $\phi = 34°$.

Answer 34.

Let B be the position of the man, D the base of the cliff, x be the distan of cliff from the ship and h + 10 be the height of the hill.

$\angle ABC = 45^\circ$ and $\angle DBC = 30^\circ$

Therefore, $\angle BDE = 30^\circ$



In ΔABC,

$\tan 45^\circ = \frac{AC}{BC}$

$\Rightarrow \frac{h}{x} = 1$

$\Rightarrow h = x$ (1)

In ΔBED,

$\tan 30^\circ = \frac{BE}{ED}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$

$\Rightarrow x = 10\sqrt{3} = 10 \times 1.732 = 17.32$

Thus, the distance of the cliff from the ship is 17.32 m.

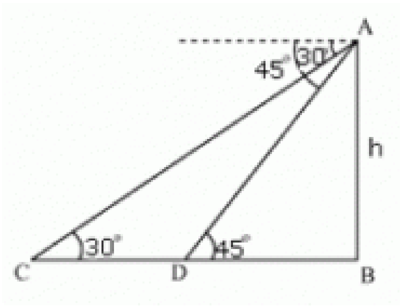
From (1),

$h = x = 17.32$

$\therefore$ Height of the cliff = 17.32 + 10 = 27.32

Thus, the height of the cliff is 27.32 m.

Answer 38.



Let AB be the tower.

Initial position of car is C, which changes to D after 720 seconds.

In ΔADB

$$\frac{AB}{DB} = \tan 45^\circ$$
$$\frac{AB}{DB} = 1$$
$$DB = AB$$

In ΔABC

$$\frac{AB}{BC} = \tan 30^\circ$$
$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$
$$AB\sqrt{3} = AB + DC$$
$$DC = AB\sqrt{3} - AB = AB\left(\sqrt{3} - 1\right)$$

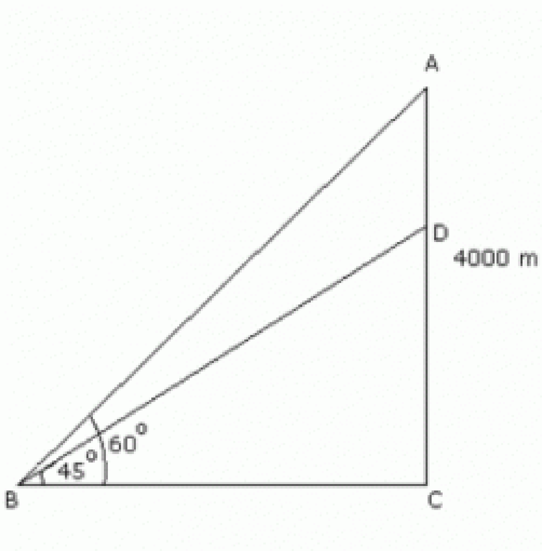
Time taken by car to travel DC distance $\left(\text{i.e.} AB\left(\sqrt{3} - 1\right)\right) = 720$ seconds

Time taken by car to travel DB distance (i.e. AB)

$$= \frac{720}{AB(\sqrt{3} - 1)} \times AB = \frac{720}{(\sqrt{3} - 1)} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$
$$= \frac{720(\sqrt{3} + 1)}{2} = 360(\sqrt{3} + 1) = 360 \times 2.732 = 983.52$$

Thus, the required time taken is 983.52 seconds = 984 seconds = 16 mins 24 secs.

Answer 40.



Let points A and D represent the position of the aeroplanes.

Aeroplane A is flying 4 km = 4000 m above the ground.

$\angle ACB = 60°$, $\angle DCB = 45°$

In ΔABC,

$$\frac{AB}{BC} = \tan 60°$$

$$\Rightarrow BC = \frac{4000}{\sqrt{3}}$$

In ΔDCB,

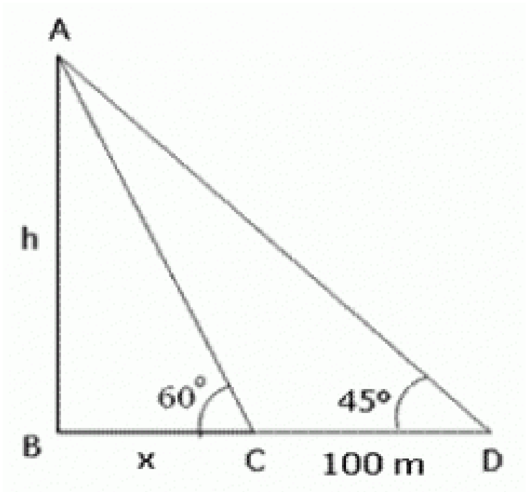
$$\frac{DB}{BC} = \tan 45°$$

$$\Rightarrow DB = BC = \frac{4000}{\sqrt{3}}$$

$\therefore AD = AB - BD$

$$= 4000 - \frac{4000}{\sqrt{3}} = 4000\left(1 - \frac{1}{\sqrt{3}}\right) = 4000 \times \frac{\sqrt{3}-1}{\sqrt{3}} = 4000 \times \frac{0.732}{1.732} = 1690.53$$

Answer 41.



Let A be the position of the parachutist and C and D be the tw observation points.

In ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$

In ΔABD,

$\tan 45° = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{h}{x+100}$

$\Rightarrow x + 100 = h$

$\Rightarrow x + 100 = \sqrt{3}x$

$\Rightarrow x\left(\sqrt{3} - 1\right) = 100$

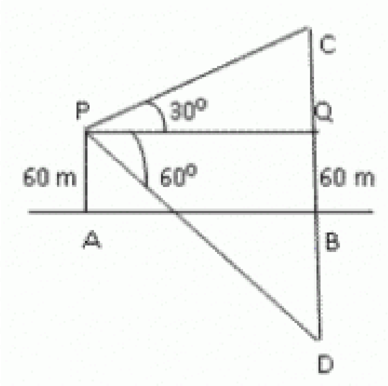
$\Rightarrow x = 100 \times \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow x = 100 \times \frac{(\sqrt{3}+1)}{3-1} = 50(\sqrt{3}+1) = 50 \times 2.732 = 136.6$

Thus, the distance of the point where he falls on the ground from tl nearest observation point (C) is 136.6 m.

Height from which the parachutist fall

Answer 43.



Let C be the cloud and D be its reflection. Let the height of the cloud is h metres. BC= BD= h

BQ=AP= 60m

$\therefore$ CQ = h-60 and DQ = h+60

In ΔCQP,

$$\frac{PQ}{CQ} = \cot 30^\circ$$

$$\Rightarrow \frac{PQ}{h-60} = \sqrt{3}$$

$$\Rightarrow PQ = \sqrt{3}(h-60) \quad \ldots (I)$$

In ΔDQP,

$$\frac{PQ}{DQ} = \cot 60^\circ$$

$$\Rightarrow \frac{PQ}{h+60} = \frac{1}{\sqrt{3}}$$

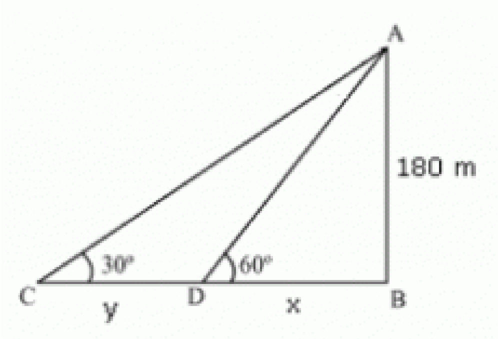
$$\Rightarrow PQ = \frac{1}{\sqrt{3}}(h+60) \quad \ldots (II)$$

From (I) and (II),

$$\Rightarrow \sqrt{3}(h-60) = \frac{1}{\sqrt{3}}(h+60)$$
$$\Rightarrow 3h - 180 = h + 60$$
$$\Rightarrow 2h = 240$$
$$\Rightarrow h = 120$$

Thus, the height of the cloud is 120m.

Answer 44.



Let AB be the lighthouse.

Initial position of boat is C, which changes to D after 2 minutes.

In ΔADB

$$\frac{AB}{DB} = \tan 60°$$
$$\frac{180}{x} = \sqrt{3}$$
$$x = \frac{180}{\sqrt{3}}$$

In ΔABC

$$\frac{AB}{BC} = \tan 30°$$
$$\frac{180}{x+y} = \frac{1}{\sqrt{3}}$$

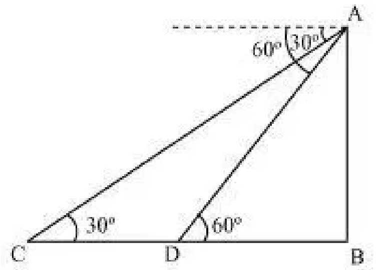
$$180\sqrt{3} = x + y$$
$$180\sqrt{3} = \frac{180}{\sqrt{3}} + y$$
$$y = 180\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 180\left(\frac{2}{\sqrt{3}}\right) = \frac{360}{\sqrt{3}}$$

Time taken by car to travel DC distance $\left(\text{i.e.} \frac{360}{\sqrt{3}}\right)$ = 2 minutes = 1 seconds

Speed of the boat $= \frac{\text{Distance}}{\text{Time}} = \frac{\frac{360}{\sqrt{3}}}{120} = \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3} = 1.732$

Thus, the speed of the boat is 1.732 m/sec.

Answer 45.



Let AB be the cliff. Then, AB = 450 m

Initial position of boat is C, which changes to D after 3 minutes.

In $\triangle ADB$

$$\frac{AB}{DB} = \tan 60^\circ$$
$$\frac{450}{DB} = \sqrt{3}$$
$$DB = \frac{450}{\sqrt{3}}$$

In $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ$$
$$\frac{450}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$450\sqrt{3} = BD + DC$$
$$450\sqrt{3} = \frac{450}{\sqrt{3}} + DC$$
$$DC = 450\sqrt{3} - \frac{450}{\sqrt{3}} = 450\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$
$$= \frac{900}{\sqrt{3}} = \frac{900}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 300\sqrt{3}$$

Time taken by car to travel DC distance $\left(\text{i.e.,} 300\sqrt{3}\right)$ = 3 minutes

Time taken by car to travel DB distance $\left(\text{i.e.} \frac{450}{\sqrt{3}}\right)$

$$= \frac{3}{300\sqrt{3}} \times \frac{450}{\sqrt{3}} = \frac{450}{300} = 1.5$$

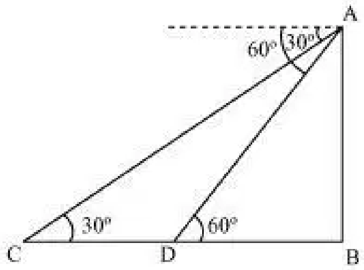
Thus, the time it will take to reach the shore is 1 min 30 secs.

Speed of the boat = $\frac{\text{Distance}}{\text{Time}}$

$$= \frac{300\sqrt{3}}{3} = 100\sqrt{3} = 100 \times 1.732 = 173.2 \text{ m/min}$$

$$= \frac{173.2}{60} \text{ m/sec} = 2.9 \text{ m/sec}$$

Answer 46.



Let AB be the tower.

Initial position of ship is C, which changes to D after 3 minutes.

In ΔADB

$$\frac{AB}{DB} = \tan 60^\circ$$
$$\frac{AB}{DB} = \sqrt{3}$$
$$DB = \frac{AB}{\sqrt{3}}$$

In ΔABC

$$\frac{AB}{BC} = \tan 30^\circ$$
$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$
$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$
$$= \frac{2AB}{\sqrt{3}}$$

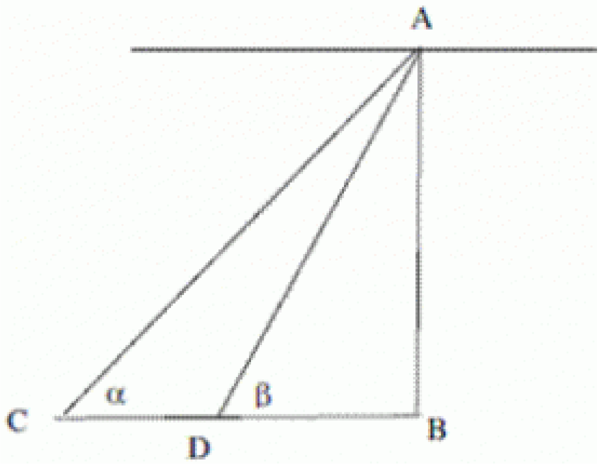
Time taken by car to travel DC distance $\left(\text{i.e.} \frac{2AB}{\sqrt{3}}\right)$ = 3 minutes

Time taken by car to travel DB distance $\left(\text{i.e.} \frac{AB}{\sqrt{3}}\right)$

$$= \frac{3}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}} = \frac{3}{2} = 1\text{min } 30\text{secs}$$

Thus, the total time taken is 3 minutes + 1 minute 30 seconds= minutes 30 seconds.

Answer 47.



In the figure, AB is the tower. A is the position of the man. C and D a the two positions of the truck.

Let the speed of the truck be x m/sec

Distance CD = speed × time = 600x

In right triangle ABC,

$\tan \alpha = \frac{h}{BC}$

It is given that $\tan \alpha = \frac{1}{\sqrt{5}}$

$BC = h\sqrt{5}$ ... (1)

In right triangle ABD,

$\tan \beta = \frac{h}{BD}$

It is given that $\tan \beta = \sqrt{5}$

$h = \sqrt{5}BD$

Now, CD = BC – BD

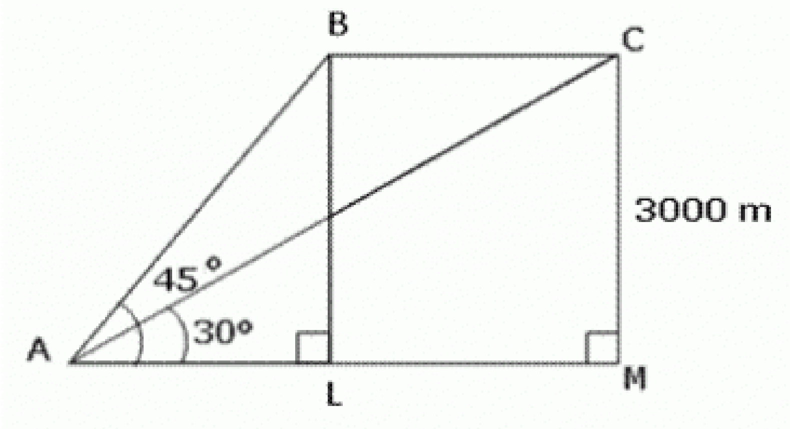
600x = 5BD – BD

BD = 150x

Time taken $= \frac{150x}{x} = 150$ seconds

Thus, the time taken by the truck to reach the tower is 150 sec = 2 m 30 sec.

Answer 48.



Let A be the point of observation on the ground and B and C be the two positions of aeroplane. Let BL=CM = 3000 m.

In ΔALB,

$$\tan 45^\circ = \frac{BL}{AL}$$

$$\Rightarrow 1 = \frac{3000}{AL}$$
$$\Rightarrow AL = 3000$$

In ΔAMC,

$$\tan 30^\circ = \frac{MC}{AM}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3000}{3000 + LM}$$
$$\Rightarrow 3000\sqrt{3} = (3000 + LM)$$
$$\Rightarrow LM = 3000(\sqrt{3} - 1)$$
$$\therefore BC = 3000(\sqrt{3} - 1)$$

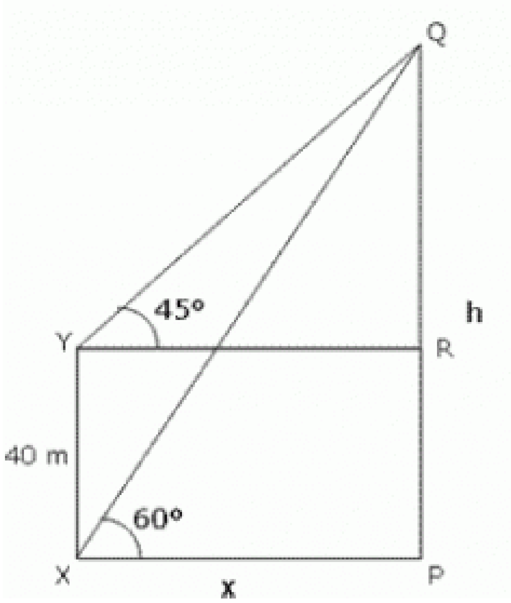
Now, time taken to travel distance BC = 15 seconds

$$\therefore \text{Speed of the aeroplane} = \frac{\text{Distance}}{\text{Time}} = \frac{3000(\sqrt{3} - 1)}{15} = 200 \times 0.732 = 146.4$$

Thus, the speed of the aeroplane is 146.4 m/sec

$$= 146.4 \times \frac{\frac{1}{1000}}{\frac{1}{3600}} \text{km/hr} = 146.4 \times 3.6 \text{km/hr} = 527.04 \text{km/hr}$$

Answer 49.



In the figure, PQ is the tower.

In ΔPQX,

$$\therefore \frac{h}{x} = \tan 60^\circ = \sqrt{3}$$
$$\Rightarrow h = \sqrt{3}x \qquad \ldots(1)$$

In ΔQRY,

$$\frac{h-40}{x} = \tan 45^\circ = 1$$
$$\Rightarrow h = 40 + x \qquad \ldots(2)$$

From (1) and (2),

$$\sqrt{3}x = 40 + x$$

$$\Rightarrow (\sqrt{3} - 1)x = 40$$

$$\Rightarrow x = \frac{40}{\sqrt{3}-1} = \frac{40(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{40}{2}(\sqrt{3}+1) = 20(\sqrt{3}+1)$$

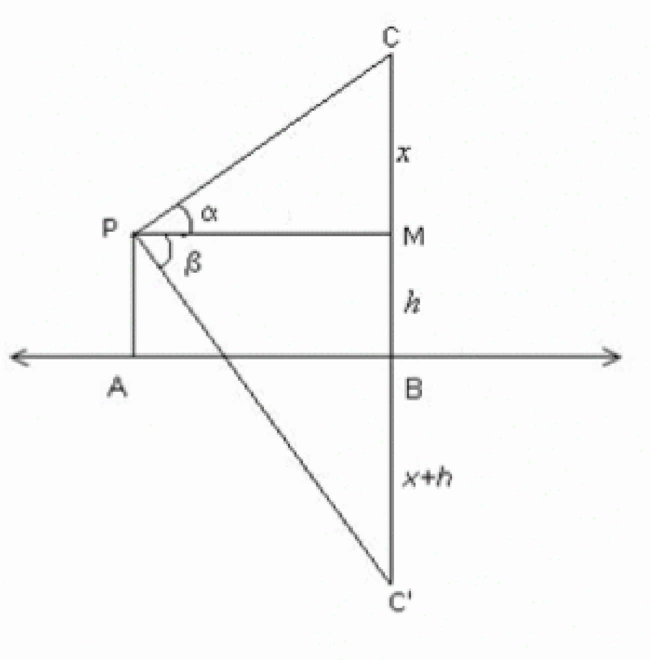
$$\therefore h = 40 + 20(\sqrt{3}+1) = 20\sqrt{3} + 60 = 20(\sqrt{3}+3) = 20 \times 4.732 = 94.64$$

Thus, the height of the tower PQ is 94.64 m.

Again, in ΔPQX,

$$\therefore \frac{h}{XQ} = \sin 60^\circ = \frac{1}{\sqrt{2}}$$
$$\Rightarrow XQ = \sqrt{2}h = 1.414 \times 94.64 = 109.3m$$

Answer 50.



Let AB be the surface of the lake and let P be an point of observation su that AP = h meters. Let c be the position of the cloud and C' be its reflection in the lake. Then ∠CPM = α and ∠MPC = β. Let CM = x.
Then, CB = CM + MB = CM + PA = x + h
In Δ CPM,

$\tan\alpha = \frac{CM}{PM}$

$\Rightarrow \tan\alpha = \frac{x}{AB}$ [∵ PM = AB]

$\Rightarrow AB = x\cot\alpha$ ... (1)

In Δ PMC',

$\tan\beta = \frac{C'M}{PM}$

$\Rightarrow \tan\beta = \frac{x+2h}{AB}$

$\Rightarrow AB = (x + 2h)\cot\beta$ ... (2)

From (1) and (2),

$x\cot\alpha = (x + 2h)\cot\beta$

$\Rightarrow x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta}$

$\Rightarrow x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = \frac{2h}{\tan\beta}$

$\Rightarrow x = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$

Again, in Δ CPM,

Answer 51.

Let P Q be h

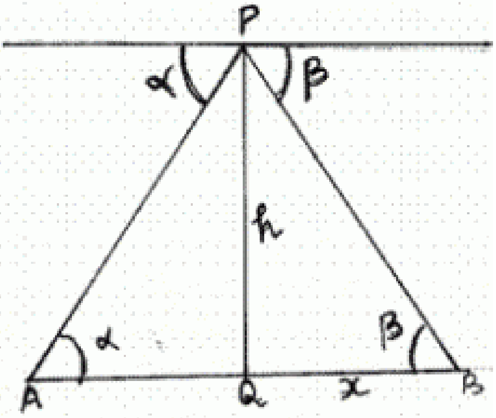
QB be x

Given : AB = 1 mile

QB = x

AQ = (1- x) mile

in ΔPAQ



$$\tan\alpha = \frac{PQ}{AQ}$$

$$\tan\alpha = \frac{h}{1-x}$$

$$1-x = \frac{h}{\tan\alpha} \quad \ldots\ldots\ldots\ldots 1$$

In ΔPQB

$$\tan\beta = \frac{h}{x}$$

$$x = \frac{h}{\tan\beta}$$

Substitute for x in equation (1)

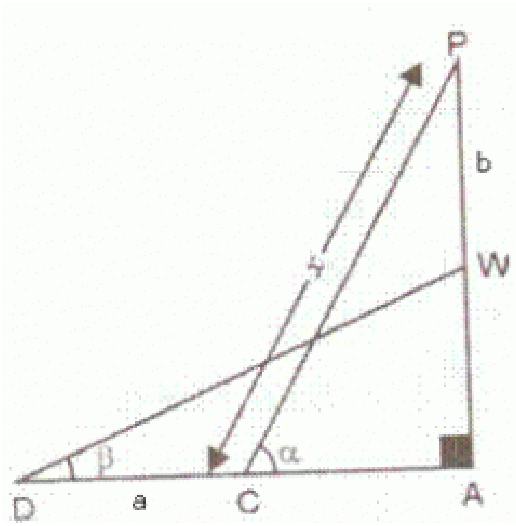
$$1 = \frac{h}{\tan\beta} + \frac{h}{\tan\alpha}$$

$$1 = h\left\{\frac{1}{\tan\beta} + \frac{1}{\tan\alpha}\right\}$$

$$\frac{1}{h} = \frac{\tan\beta + \tan\alpha}{\tan\beta\tan\alpha}$$

Thus, the height in miles of aeroplane above the road is $\frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$.

Answer 52.



Let CP and DW be the two positions of the ladder such that CP = DW = (say).

CD = a, PW = b, ∠ACP = α and ∠ADW = β

In ΔAPC,

$\frac{AC}{CP} = \cos\alpha \Rightarrow AC = x\cos\alpha$ ...(i)

In ΔADW,

$\frac{AD}{DW} = \cos\beta \Rightarrow \frac{AC+CD}{DW} = \cos\beta$

$\Rightarrow \frac{x\cos\alpha + a}{x} = \cos\beta$ [using (i)]

$\Rightarrow x = \frac{a}{\cos\beta - \cos\alpha}$ ...(ii)

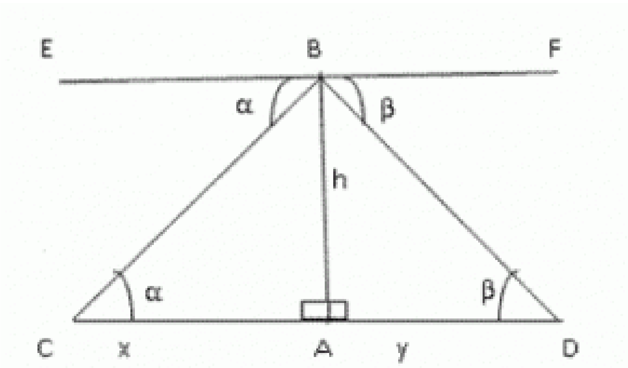
Again in ΔAPC, $\frac{AP}{CP} = \sin\alpha$

$\Rightarrow AP = x\sin\alpha = \frac{a\sin\alpha}{(\cos\beta - \cos\alpha)}$ ...(iii) [Using (ii)]

Again in ΔADW, $\frac{AW}{DW} = \sin\beta$

$\Rightarrow AW = x\sin\beta = \frac{a\sin\beta}{(\cos\beta - \cos\alpha)}$ ...(iv)

Answer 53.



Let AB be the lighthouse of height h m. Let AC = x and AD = y.

In ΔCAB,

$$\frac{AB}{AC} = \tan \alpha$$

$$\tan \alpha = \frac{h}{x}$$

$$x = \frac{h}{\tan\alpha} \quad \dots (i)$$

In ΔDAB,

$$\frac{AB}{AD} = \tan\beta$$

$$\tan \alpha = \frac{h}{y}$$

$$y = \frac{h}{\tan\beta} \quad \dots (ii)$$

Distance between the ships = x+y

$$= \frac{h}{\tan\alpha} + \frac{h}{\tan\beta}$$

$$= h\left(\frac{\tan\beta + \tan\alpha}{\tan\alpha \tan\beta}\right)$$